Lab 12

Center of Mass

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Learning Objectives

- Relate center of mass to center of gravity
- Use the position and mass of one or more objects to determine the center of mass
- Apply center of mass to stability

Introduction

If you have ever tried slack lining, you know how difficult it can be to keep your balance. What you might not have thought about is that waving your hands around and shifting your weight is a way of changing your center of mass. By keeping your center of mass in the right position above the line you prevent yourself from toppling over.

Center of Mass

Center of mass is the point in space where all the mass of an object balances. No matter what the object's shape or how it is moving, the center of mass moves as if all the mass of the object were concentrated at that point.

Finding the center of mass for objects that have mass distributed **uniformly** can be pretty simple. For example, the center of mass of a long metal rod is exactly in the center of the rod. For a square sheet of paper, the center of mass is in the center of the square. One way to think about center of mass is to imagine where the object would balance if set on a single point. If you try to balance an object at a point where more mass exists in one region surrounding that point than another, the object will tip in that direction. In order for the object to stay at one specific orientation, the forces due to gravity on all sides of the object must cancel each other out. For objects with their mass distributed evenly, the center of mass is located at the **geometric center**. This does not necessarily have to exist on the object itself; a donut's center of mass is located within its empty interior.

? Did You Know...

Professional bowlers make the ball curve into the pins at just the right moment by utilizing the ball's center of mass. A ball with a center of mass just slightly removed from the geometric center will tend to rotate in one direction down the lane, making it much easier to curve into the pocket!

Center of Gravity

Since the center of mass of an object can be found based on where it balances, the center of mass is often called the **center of gravity**. This is the point where you would draw the arrow for the force due to gravity on a free body diagram of an irregular object.

Imagine that you and a friend are on opposite sides of a seesaw; if you both weigh the same, the seesaw will balance perfectly in its center. The forces of gravity on each side balance each other out, and the center of mass is in the middle of the seesaw. If another person gets on your side with you, the seesaw will fall in your direction. You would have to move the center of the seesaw closer to your side to stay balanced. When more mass is added, the center of mass moves toward the area where most of the mass is located.

Finding Center of Mass

Knowing the center of mass of a complicated object can make solving certain problems fairly simple. Since the center of mass moves as if all the mass exists at that one point, you can treat complicated objects as a point object of the same mass located at the center of mass. For instance, if you throw a hammer the center of mass still follows a parabolic trajectory. All points on the hammer rotate about the center of mass, giving you the illusion that the hammer is moving back and forth in the air—but really its center of mass is moving along the same line as a ball thrown at the same speed.

So how do you find the center of mass? For a uniform object, the center of mass will be at the geometric center. However, for irregularly shaped objects things get a bit more complicated. The simplest method to find the center of gravity is to try to balance the object on a point, such as balancing a spoon on your fingertip. Mathematically, the center of mass of a system can be found by adding up all the individual masses, m, at specific positions, x, and diving by the total mass of the system, M:

$$x_{CM} = \frac{\sum m_i x_i}{M} \qquad y_{CM} = \frac{\sum m_i y_i}{M}$$

Here, $\sum m_i x_i$ is the sum of individual points with mass, m, and position, x, and M is the total mass of the system. In the case where there are only two masses the first equation takes the form:

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{M}$$

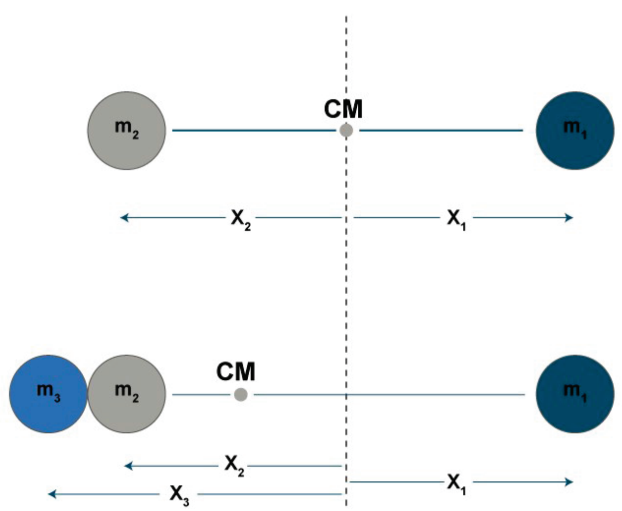

***Figure 1:* Position and mass of objects affect center of mass.**

The center of mass for a set of objects with masses m_1 and m_2 is in the center of the space between them when the masses are equal (Figure 1). The distances x_1 and x_{-2} are measured relative to an arbitrary point, usually the origin at (0,0). Adding a third mass on the left shifts the center of mass in that direction. You can imagine the center of mass as being located where the set of objects would balance when placed at a single point. In three dimensions, the x, y and z directions all must be taken into account.

Stability

The location of the center of mass of an object will tell you a lot about how **stable** that object is when standing on one

Figure 2: ***The Leaning Tower of Pisa.***

side. Imagine an object standing upright on a flat surface. If the center of mass is situated somewhere over the base of the object, it will not suddenly tip over. The reason lies in the position of its center of gravity – which is still located above its base. If you change the object or the surface it is standing on so that the center of mass is located *outside* its base, the object will tip over in that direction. This explains why the Leaning Tower of Pisa doesn't tip over (Figure 2). In general, the closer an object's center of mass is to the ground, the more difficult it is to push the object over.

Many athletes use the idea of center of gravity to improve their performance. Athletes who understand how to alter their center of mass to adapt to each situation are usually very successful. For example, football players bend down, take a wide stance, and shift their weight forward when they prepare to block and tackle. What they are actually doing is lowering their center of gravity and creating a wider base of support to improve their stability.

Taller objects with high center of mass are less stable than shorter objects with a high center of mass, especially when located on an incline (Figure 3). The location of center of mass also is very important for engineers who design cars and other vehicles. If the center of mass is too high (as with some sport utility vehicles) the car will more easily tip over on a sharp turn. Sports cars are designed to have a very low center of mass, which allows them to make turns at incredible speeds without tipping. The following experiments will help you practice identifying the center of mass and show you some ways to calculate it more precisely.

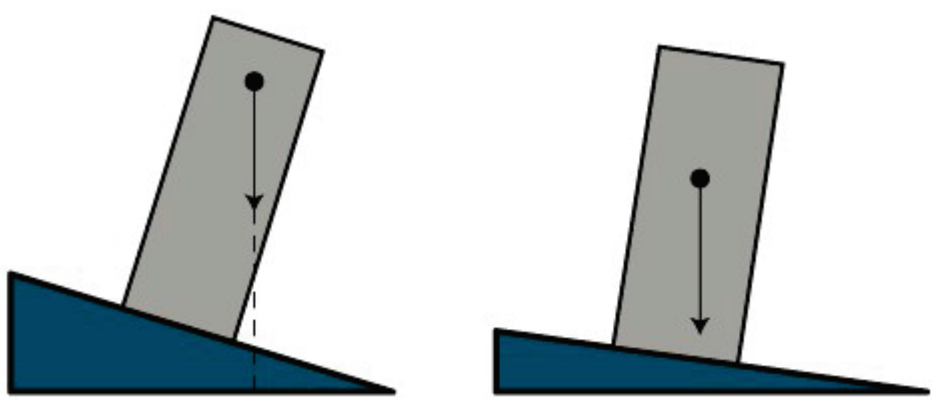

Figure 3: ***The rectangular blocks both have a high center of mass, as indicated by a black dot. The arrow represents the direction of the force of gravity on the center of mass.***

Pre-Lab Questions

1. Estimate where the center of mass is in the objects pictured below. Explain why.

a.

b.

c.

d.

2. Explain why the object in Figure 4 does not represent a realistic object and center of mass.

***Figure 4:* An object and a center of mass.**

3. Consider three masses, m_1 = 250 g, m_2 = 100 g, and m_3 = 50 g, hanging on a ruler (Figure 5). The origin is located at the center of the ruler (x = 0). Mass 1 is located at -6.5 cm and mass 2 is located at 9 cm. At what location does Mass 3 need to be in order for the center of mass of the system to be located at the origin?

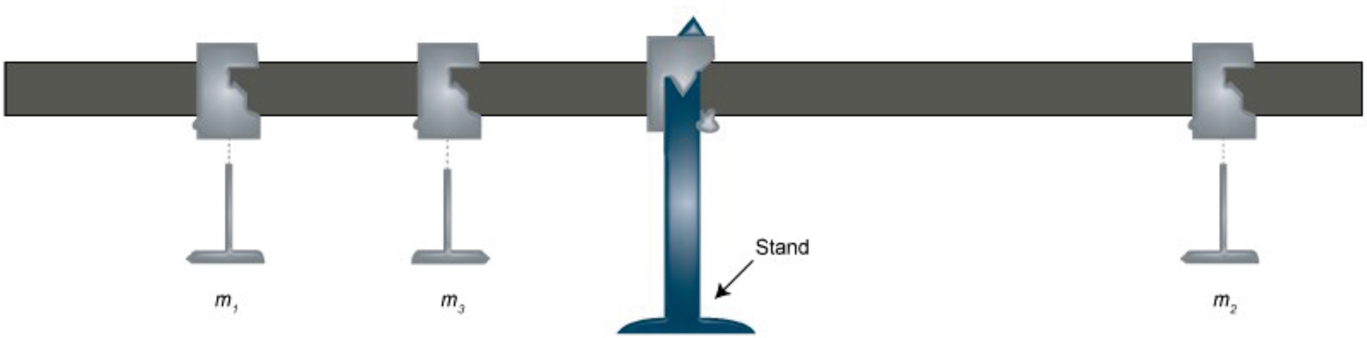

***Figure 5:* Three masses hanging on a ruler.**

Experiment 1: Stability

In this experiment you will test the stability of a stack of blocks as the angle of the surface the stack rests on gradually increases. Using a fishing sinker tied to the center of mass, you will be able to see the position of the center of mass relative to the base of the object as it begins to tip over.

Materials

1 Fishing Sinker
Protractor
30 cm. String
Masking Tape
Ramp Runway
Ruler

4 Wooden Blocks
*1 Empty Water Bottle with Lid
*Water

*You Must Provide

Procedure

Part 1

1. Mark the location of the center of gravity on one side of a wooden block with a piece of masking tape (the middle).
2. Using masking tape, attach one block above and below your original so that your center of gravity mark is visible, making a 3-block-high tower.
3. Use a ruler to measure and cut 30 cm. of string. Tape the string to the mark on the middle block with 20 cm hanging downward.
4. Attach a sinker to the end of the string. Set the block stack on top of the ramp, and line the edge of the ramp runway up with the edge of a table so that the string can dangle.
5. Increase the incline of the ramp runway, and notice the relationship between when the block stack starts to tip over and the location of the string. Record your observations in Table 1.
6. Try this out with four blocks stacked. Make sure to move your center of mass to the middle of the tower (between the second and third blocks). Record your observations in Table 1.

Table 1: Block Observations

Block Arrange-ment	Observations
Three Stacked Blocks	
Four Stacked Blocks	

Part 2

1. Fill one of the water bottle one fourth full of water.
2. Stand the bottle upright on a table. Slowly tilt the bottle by pushing the top of one side with one hand while supporting the opposite side of the bottle with a finger form your other hand.
3. Push the top of the bottle further and further checking every once in a while to see if the bottle will fall back upright if you were to stop pushing.
4. Continue pushing the top of the bottle up until an angle where the bottle will not tip back upright and support that angle with the finger form your other hand.
5. Use the protractor to measure the angle between the side of the bottle your finger is supporting and the table. Record the angle in Table 2.
6. Repeat Steps 2 - 5 with the bottle half full, three fourths full and completely full. Record your angles in Table 2.

Table 2: Angle Just Before Bottle Tips Over

Amount of Water in Bottle	Angle (°)
$^1/_4$ Full	
$^1/_2$ Full	
$^3/_4$ Full	
Full	

Post-Lab Questions

1. When did the blocks typically fall over?

2. Which stack of blocks (3 or 4) had a lower center of mass? Which set tipped over at the largest angle? How do you know?

3. If you were building a skyscraper in a windy city, where would you want most of the building's weight to be located?

4. Draw a rough diagram for each case showing the placement of the center of mass (point CM) and the maximum angle of the bottle reached.

5. Explain why you were able to tilt the bottle more in some cases more than others.

6. How soon do you think the bottle would tip over if you could fill only the *top half*?

Experiment 2: Irregular Shapes

In this experiment, you will determine the center of mass for objects that have an irregular shape.

Materials

Hole Punch	Ruler
Kit Box	Scissors
30 cm. String	2 Washers
Permanent Marker	
Printer Paper	*You Must Provide
1 Push Pin	

Procedure

1. Use the scissors to cut an irregular shape out of a piece of paper. Any shape will work!
2. Cut a 30 cm length of string and tie one metal washer to each end. This will function as a "plumb-bob" that hangs down as a vertical line.
3. Set one side of the physics kit box flush with the edge of a table and stick a push pin in the cardboard near the top (Figure 6).

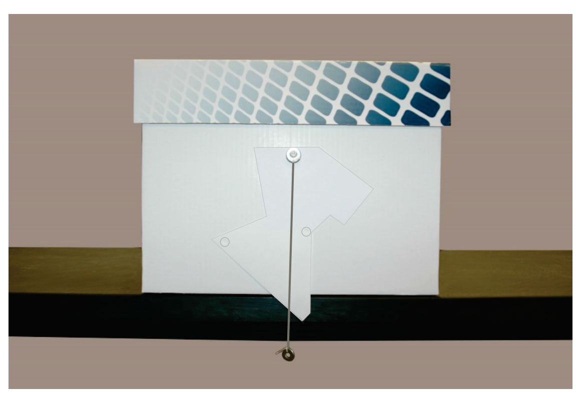

***Figure 6:* Step 3 reference.**

4. Punch a hole in three different spots around the edge of the shape, but not too close together.
5. Hang the shape through one of the three holes on the push pin making sure the shape can move freely
6. Hook the plumb-bob to the push pin with the washer.
7. Note how the string hangs across the shape. Make a mark on the side of the shape opposite the hole in line with the plumb-bob string. Use this mark to draw a straight line through the shape, from the hole to your mark.
8. Take the shape and plumb-bob off the pin, and switch to a new hole on the shape. Repeat Steps 5 - 7 until you have three lines drawn on the shape.

Post-Lab Questions

1. What do you notice about the lines you drew?

2. What does the point where the three lines intersect represent?

3. When you hang the shape from the pin, it balances around that point. What does this tell you about the distribution of mass about this line?

4. Is the third line necessary to find the center of mass?

Experiment 3: Center of Mass of a Hanging Mass System

In this experiment you will determine the center of mass of a hanging mass system.

Materials

50 g Mass
100 g Mass
250 g Mass
Masking Tape
Mirror Support
3 Paper Clips
Ruler
(1) 8 oz. Styrofoam Cup

Procedure

1. Find a flat stable surface such as a table or floor.
2. Place the cup upside down on the surface.
3. Place the mirror support on top of cup and secure each side of the base with one piece of masking tape
4. Take the ruler and balance it on the mirror support (Figure 7). As soon as you get the ruler to balance, record the location in Table 2 as the origin.

Figure 7: ***System set up.***

5. Use three paper clips to fabricate mass hangers that will hold the masses. The easiest way to do this is to bend the outer wire through the inner loop of the paper clip and then pull them down together (Figure 8).
6. Holding the ruler above the table, slide on the three mass hangers.
7. Still holding the ruler with one hand, place the masses onto the mass hangers.

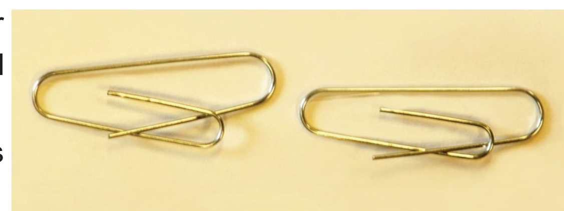

Figure 8: ***Paper clips molded into mass hangers.***

8. Once the masses are hanging, slide the 250 g mass to 6.5 cm left of the origin and the 100 g mass 9 cm to the right of the origin. Record the location of the masses on the ruler in Table 3. For example, if the origin is located at 16 cm, the 250 g mass is located at 9.5 cm.
9. Place the 50 g mass at the location you calculated in Pre Lab Question 3.
10. Now set the origin of the ruler on the mirror support and slowly let go of the ruler to see if your system of hanging masses balances.
11. If it does not balance perfectly, adjust the position of the 50 g mass to get the system to balance.
12. Once the system is balanced, record the location of the 50 g mass on the ruler in Table 3.

Table 3: Location of Masses on Balanced Ruler

Origin (cm)	m_1 (g)	x_1 (cm)	m_2 (g)	x_2 (cm)	m_3 (g)	x_3 (cm)

Post-Lab Questions

1. Fill out the following Table 4 and 5 to calculate your percent error for the experiment.

$$\text{Percent Error} = \frac{|(\text{Experimental - Actual})|}{\text{Actual}} \times 100\%$$

Table 4: Location of Masses Balanced as Calculated from the Origin

Mass (g)			
Distance from Origin (cm)			

Table 5: 50 g Mass Data

$x_{3,\ theoretical}$ (cm)	$x_{3,\ experimental}$ (cm)	Percent error

2. How does your prediction compare to the actual center of mass? Explain.